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2022-2023

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Increasing Confidence in Mathematics by Grading the Process and not the Product

The Mathematics Process over The Mathematics Product

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Goals and Objectives

The two major goals of this project are as follows: to build students' confidence in mathematics by assessing the process of their assessments and not solely the product of their assessments; and to increase student's abilities to think mathematically and critically. Research shows that a teacher can always find something positive to convey to a student when the process of their work is examined instead of the product of their work.

Students will learn that there is not always one approach to arrive at an acceptable conclusion in mathematics. They will build confidence in their individual conclusions and develop skills that foster critical thinking. Students in The Process over The Product classroom are encouraged to be in constant dialogue with the teacher, themselves and other students over the viability of their processes and conclusions.

One objective is to get students accustomed to expecting questions about their conclusions and then justifying or finding support to justify those findings. The dialogue that is created in the classroom serves as a mechanism to foster reflection, encourage doubt, and then to build confidence.

In an Algebra I class students are expected to carefully demonstrate how they would solve a three step equation. The steps in the process are just as valuable as the product at the end of the process. In a calculus class a student would be expected to explain and justify in detail her choice of a particular method of differentiation. Dialogue in conversation and in writing is one of the dominant attributes that is developed in students who are in a Process over The Product classroom.

Florida Standards

912.A-APR.1.1
Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

912.F-BF.1.1b
Combine standard function types using arithmetic operations.

LAFS.910.L.1.1 Demonstrate command of the conventions of standard English grammar and usage when writing or speaking.

LAFS.910.L.1.2 Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing.

Course Outline/Overview

The diversity (race, culture, emotional, cognitive, etc) that currently exists in Public School classrooms demands that many traditional standards of student assessments must be abandoned if widespread validation and affirmation of all students is to take place. This level of affirmation and validation can be accomplished without compromising fair and equitable standards of excellence. Simply by entering a classroom, certain demographic groups may be at a distinct academic disadvantage. This academic disadvantage is not correlated to race or culture but is often linked to historic inequities which manifest themselves in poverty.

According to Julia Isaacs (Brookings Institute, 2012), “Poor children in the United States start school at a disadvantage in terms of their early skills, behaviors, and health. Fewer than half (48 percent) of poor children are ready for school at age five, compared to 75 percent of children from families with moderate and high income, a 27 percentage point gap.” Without intervention schools then become factories that simply reproduce poverty.

Teachers must ensure that their classrooms are not factories that perpetuate and reproduce poverty and different classes of people. On the contrary, public schools must be a place where each child is ensured an equitable opportunity to succeed. How students are assessed can go a long way towards cultivating a classroom environment where there is sufficient space for each student (rich and poor) to be affirmed and and to be validated.

Specifically in the mathematics classroom, students are often driven by a need to find a solution or an answer; they are essentially interested in the final product. This is not entirely the fault of the student; teachers and the educational system (obsession with data points) itself are often the drivers of this madness. The fixation on the final product, in my experience, drives students further away from the actual academic prize, which is comprehending the process.

The idea behind The Process over The Product Grading system in my mathematics classroom is simple: “Grade the entire process of every assessment item.” On the surface this may seem exhausting, but once you have established your process over the product assessment system the benefits to both student and teacher will be immeasurable.

Teachers must first cherish and then communicate a commitment to process in their lesson planning, instructional activities, and classroom conversations. Students will then begin to value the process themselves and this will become evident in student’s assessment responses. Below is how John, a fictitious student, would fare under a process based assessment system compared to a product based assessment system:

Sample assessment question graded from a product perspective and from a process perspective.

Question 1. Solve the following equation for x: $2x^{2} - 12x + 4 = x^{2} - 3$	Product Based Assessment For John’s Response	Process Based Assessment For John’s Response
	$2x^{2} - 12x + 4 = x^{2} - 3x$ Subtract x^{2} from both sides $x^{2} - 12x + 4 = -3x - 4$ Add $3x$ to both sides $x^{2} - 9x + 4 = -4$ Add 4 to both sides $x^{2} - 9x + 8 = 0$ Factor to find the zeros $(x - 9)(x + 8) = 0$ Solve $x - 9 = 0 \; and \; x - 8 = 0$ $x = 9 \quad and \quad x =- 8$ *John is not able to solve for x because he has forgotten how to factor the quadratic equation, so he stops after the first three steps.* In a product based system, especially a multiple choice assessment, this response would be graded as incorrect. John's score on this question would be: $\frac{0}{5}$ ✖	$2x^{2} - 12x + 4 = x^{2} - 3x - 4$ Subtract x^{2} from both sides $x^{2} - 12x + 4 = -3x - 4$✓ Add $3x$ to both sides $x^{2} - 9x + 4 = -4$✓ Add 4 to both sides $x^{2} - 9x + 8 = 0$ ✓ Factor to find the zeros $(x - 9)(x + 8) = 0$ $(\frac{1}{2}$✓$)$ Solve $x - 9 = 0 \; and \; x - 8 = 0$ $x = 9 \quad and \quad x =- 8$ The teacher writes the following: “John, Good work on simplifying the equation. Think about the following: “what are the factors of 8 when you multiply those factors you will get 8, but when you add them you will get -9. Good job, thus far, see me after class so we can discuss. In the process based system, John receives the following score: $\frac{3.5}{5}$

Think About It

If this was a multiple choice exam with 10 similar questions, John would likely score a 0/50 (0% or an F) using the product based assessment system. Conversely, in the process based assessment system, John would likely score a 35/50 (70% or a C). John is aware that he did a portion of the questions correctly. Validating and affirming him for his work would lift his self esteem and position him emotionally to better comprehend factoring. There are many Johns in our classroom.

Here is the justification, in granular detail, for John's score of 3.5/5 on this particular assessment item

- John recognizes that this is an equation and not an expression.
- He recognizes and knows what like terms are and what he can do to combine them.
- John inherently knows that he can only combine variables that are raised to the same power.
- He knows that when he combines these like variables he is only adding their coefficients.
- John knows that he can combine constant terms.
- He understands the rules of equality.
- He understands that if he adds a term on the left that he must also add a term on the right; the addition property of equality. (John adds 3x on the right and then adds 3x on the left; he also adds 4 on the right and then adds 4 on the left..)
- John understands that if he subtracts a term on the right he must also subtract that same term from the left; the subtraction property of equality. (John subtracts x^2 from the right and also subtracts x^2 from the left.
- John's process is very efficient because he ensures that the leading term, x^2, remains positive.
- Although John has factored the quadratic expression on the left incorrectly, he knows that he must have two factors (x-9)(x+8) = 0
- By setting the factors equal to zero and then solving can imply that John understands that the x values are solutions of this quadratic equation.
- It can also be inferred that he understands that these x values are also x-intercepts.
- John correctly sets these incorrect factors to zero and then solves them.
- John knows a whole lot about this problem. The only thing in John's process that is entirely incorrect is the final result or the product of his work; John's entire process is correct!

$2x^2 - 12x + 4 = x^2 - 3x - 4$

Subtract x^2 from both sides

$x^2 - 12x + 4 = -3x - 4$✓

Add $3x$ to both sides

$x^2 - 9x + 4 = -4$✓

Add 4 to both sides

$x^2 - 9x + 8 = 0$ ✓

Factor to find the zeros

$(x - 1)(x - 8) = 0$ ********John can be taught to factor the expression and then solve the equation in this manner.

$x - 1=0$ and $x - 8 = 0$

$x = 1 \ and \ x = 8$

Tips for a successful Process Based Assessment System

- Get to know your student.
- Get to know your content area intimately.
- Reflect on individual student academic needs as you lesson plan.
- Ensure that math lessons show a connection between math topics (students are likely to remember previous content when it is connected to current content).
- Constantly connect the smaller ideas of math to the bigger ideas of math and then connect the bigger ideas to the student’s real life and to the world.
- Construct assessments as you lesson and unit plan.
- Ensure that assessment items are valid by deriving them from the lesson or unit standards and objectives.
- Ensure that the lesson and unit objectives are clearly met
- Review assessment data, over time and over periods, to ensure that assessment items are reliable
- Discuss all assessment items with students, especially outlier items, post test. This will provide insight for future test construction
- Create your own assessment items; this increases knowledge of the content and intimacy with the content.
- Explode each assessment item prior to the assessment to ensure that you know all of the steps (process) and permutations to a possible solution.
- Be aware that a student can also present a process and a solution that you did not consider
- Assign each step of the solution process a fraction of the item’s total score.
- Grade the entire process and not only the solution
- Assess orally during classroom discussions and create a culture of process over product
- Openly encourage different paths to solution
- Weaponize doubt in order to encourage and to enhance critical-mathematical thinking.

Multiplying Polynomials Process Based Learning Plan- Cooperative Learning (Teams-Games-Tournaments)

Name: Kelsey Major
Date: 7/31/2021

Subject Area(s): Algebra II Grade Level(9th and 10th): Allocated Time: **90 minutes** Topic: Content Areas

Addressed : Multiplying Polynomials :Math✔ English/Lang. ✔Arts Science Technology✔

___SS ___Other ___________________

All lesson plans address the Content Area of English/Language/Arts.

Context [1]: This is an advanced group of honors students who have taken geometry the previous year, so algebra concepts have to be re-introduced. The group does contain three students who have 504 Plans; these students will be afforded the extra time for assessments and homework per their 504 plan.
Standards: 2 State Standards (Florida Standards) for each content area marked above. 912.A-APR.1.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials; 912.F-BF.1.1b Combine standard function types using arithmetic operations; LAFS.910.L.1.1 Demonstrate command of the conventions of standard English grammar and usage when writing or speaking.; LAFS.910.L.1.2 Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing.
Instructional Objectives/Learner Outcomes: The students will be able to multiply polynomials. They will be able to multiply monomials; monomials and binomials; and binomials and binomials. Students will be able to determine if polynomials are closed under multiplication.
Assessment: ***The assessment will be graded based on the process of the students' work. The students will have an ongoing informal assessment as a part of the classroom dialogue during the lecture segment. Students will be informally assessed during the team game segment. During the tournament segment, students will be assessed individually (formative assessment); team scores will be combined and the winning team will earn a reward. The final assessment will be a "relative summative" assessment and the scores from the individual teams will be averaged. The team with the highest average and with the greatest comparative improvements will receive bonus points.***

Materials/Equipment/Resources: (List all materials, equipment, and resources needed for both the teacher and the students (K-12).) *Materials needed for teacher: Smart Board, Internet, Computer Based Textbook; Whiteboard; Blank Paper; Permanent Markers; Dry Erase Markers; Google Sheets; Khan Academy. Materials needed for students (K-12): Laptop or Notebook computer; notebook, pencil, sharpener, Internet Access, digital textbook; Khan Academy; Google Sheets; smartphones.*

Vocabulary: Polynomial, Binomial, Trinomial, Monomials, Operations, Variables, Product, Exponents, Like

Terms

Instructional Procedures:

Introduction 15-Minutes

- Polynomials are everywhere in mathematics and in our world. You know how to name and how to order them. Today you will learn how to manipulate polynomials with a familiar operation.
- You already know the operations that we are going to use to manipulate these polynomials, namely, multiplication.
- The curves on your favorite amusement park rides are modeled (by engineers) using polynomials; Engineers use them to design bridges; and the cost of your next Uber or taxi cab ride is calculated based on a polynomial formulation.
- Students will use the following technology: Khan Academy; Google Sheets; laptops; smart board; and your laptops or smartphones.

Step-By-Step Procedures Interactive Lecture 15-Minutes

- Teacher begins the lecture by multiplying two monomials and establishes basic rules that do not change: Example 1(The bases of the variables are the same) - $(3m^{2})\ by\ (2m^{4})$. A student is asked and is helped to perform one of the following steps with peers and or teachers assistance if necessary.

1. Teacher explains: The commutative property of multiplication allows coefficients and variables to be reordered.
2. (3)(2)$(m^{2})(m^{4})$
3. Coefficients are multiplied together: 6$(m^{2})(m^{4})$
4. If the bases of the variables are the same then add their exponents: $6m^{2+4} = 6m^{6}$
   - Multiplying monomials with different variable bases: Example 2 - $(-\ 3y^{3})(5x^{7})$
   - Reorder the expression : $(-\ 3)(5)y^{3}x^{7}$
   - Multiply the coefficients: $((-\ 15)y^{3}x^{7}$
   - The variables cannot be combined because they are not the same. The product of the monomials is: $(-\ 15)y^{3}x^{7}$
   - Ask students if the product is still a polynomial by definition (yes). "Polynomials when multiplied by other polynomials always produce another polynomial. Therefore polynomials are closed under multiplication."

- Three students are chosen to simplify the following monomial on the whiteboard with help from their peers:
  $(3x^{6})(4x)$ $(-\ 3xy)(2x^{2}y)$ and (5xy)(-4p)
- Multiplying monomials and binomials (explain that this is achieved with the distributive property). A student is asked and is helped to perform one of the following 7 steps with peers and or teachers assistance if necessary.
  1. **Example 3** - $2m^{2}(3mx - 12m^{3})$
  2. Best practice is to rewrite as: $2m^{2}(3mx + (-\ 12m^{3}))$
  3. Apply the distributive property: $2m^{2}(3mx) + 2m^{2}(-\ 12m^{3})$
  4. Reorder: $(2)(3)m^{2}mx + 2(-\ 12)m^{2}m^{3}$
  5. Multiply coefficients: $6m^{2}mx - 24m^{2}m^{3}$
  6. Add the exponents of like variables (if no exponent is present, the exponent is 1):

$6m^{2+1}x - 24m^{2+3}$

7. Note that the two terms cannot be subtracted because they are not identical (the second term does not have an x: $6m^3x - 24m^5$
8. Ask students if the product is still a polynomial by definition (yes). "Polynomials when multiplied by other polynomials always produce another polynomial. Therefore polynomials are closed under multiplication."

- Multiplying a binomial and a binomial. A student is asked and is helped to perform one of the following 7 steps with peers and or teachers assistance if necessary.
   1. **Example 4 -** $(3x - 5)(2x^2 + 2)$
   2. Use the FOIL method: First, Outer, Inner, and Last
   3. The algorithm: Multiply first (F) terms in each binomial: $(3x)(2x^2) = 6x^{1+2} = 6x^3$
   4. Multiply the outer (O) terms: $(3x)(2) = 6x$
   5. Multiply the inner (I) terms: $(-5)(2x^2) = -10x^2$
   6. Multiply the last (L) terms: (-5)(2) = -10
   7. Add the products together and list in descending order: $6x^3 - 10x^2 + 6x - 10$
   8. Ask students if the product is still a polynomial by definition (yes). "Polynomials when multiplied by other polynomials always produce another polynomial. Therefore polynomials are closed under multiplication."
   9. Multiplying a binomial and a monomial with variable exponents. A student is asked, with help if necessary, to perform one of the following 7 steps.
- **Example 5 -**
   1. $(3x^t + 7)(-5x^{2t})$
   2. Apply the distributive property: $-5x^{2t}(3x^t) + (7)(-5x^{2t})$
   3. Reorder: (-5)(3)$x^{2t}x^t + (7)(-5)x^{2t}$
   4. Multiply coefficients: $-15x^{2t}x^t - 35x^{2t}$
   5. Add the exponents of like variables for each monomial: $-15x^{2t+t} - 35x^{2t}$
   6. Simplify exponents: $-15x^{3t} - 35x^{2t}(\mathit{the\ monomials\ cannot\ be\ combined\ because\ the\ exponent\ on\ the\ varial}$
   - Ask students if the product is still a polynomial by definition (yes). "Polynomials when multiplied by other polynomials always produce another polynomial. Therefore polynomials are closed under multiplication."

Full lesson plan with the Team-Games-Tournament can be found here: https://docs.google.com/document/d/1E8FugkPNEzrcNO9p_d2nV1kPNL6IoUy-AXT9Dx_hhck/edit?usp=sharing

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